A method of product family design based on the theory of polychromatic sets

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Abstract. In order to deal with the complicated process and multiple solutions of reconfiguration confronting the product users in the mass customization, polychromatic sets are put forward to conduct the product configuration. Based on the relationship between customer needs and product series, on the relationship between function needs and modules, and on the relationship between capability needs and modules, the multilayer polychromatic set contour-comprising matrices are established on the function sets of the product. The unified colour reasoning algorithm is proposed in each layer and the solving process of product configuration is established based on the analysis of product configuration characteristics, and the result of product configuration is expected to be improved by combining constraint matrix with the above mentioned algorithm. More importantly, an applied case of identification system is provided to verity the reliability and rationality of the polychromatic set solving algorithm in the product configuration of product family design.

Key words. Product family, polychromatic sets, product configuration, unified colour, constraint matrix.

1. Introduction

The purpose of mass customization is the quick response and satisfying customer's personalized requirements. Meanwhile, the product design is aimed at product family; it has changed from a single-oriented product to series-oriented one in circumstances of mass customization [1]. Since the product family can save both your time and cost, in order to meet customers' needs for different product functions and features by reusing common structure and technology in the similarity

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product, a standardized and well-considered product platform in the product family is indeed required.

Recently, the main research of product family is focused on the platform design and the product configuration. Yang Gang et al. configurate template on product family by parameter, which can realize mass customization [2]. Xu Zhijia proposed a theory based on Stackelberg, and co figurate product family by restrict of low carbon [3]. Gao Xinqin in order to meet the needs of configuration diversification and configuration, the model scalability polychromatic sets are used to conduct the product configuration. But this method lacks the description in mathematics and arithmetic flow; it will cause difficulty to the engineers' understanding and their programming [4].

Polychromatic sets are a new method for building model of producing process; it can simulate different targets with the same math model, such as the concept design [6,7] the product assembly[8,9] and so on. Polychromatic sets can be used to describe the relationship among different features of complex machines and can be convenient for programming and being well applied to the complex systems [10, 11]. In a word, the method of Polychromatic sets is used to design for the mass customization and to improve the efficiency of the decision-making; it can help to make the process of choosing projects standardized and formulaic in the mass customization, and it is easy to be programmed and calculated since the author has already built the mapping between the model of product family and customer's requirements.

2. The concept of Polychromatic sets

The element a_i of sets $A = (a_1, ..., a_i, ..., a_n)$ only represents a name in traditional sets, which doesn't include any attributes. The 5th reference puts forward a concept, named as the polychromatic sets, which can remedy the defects mentioned above. Both the elements and the sets themselves can be wiped in different colours, in order to represent the property of research objects and that of their elements. As for the polychromatic sets$A = (a_1, ..., a_i, ..., a_n)$, the colour sets $F(a_i)=(f_1, ..., f_i, ..., f_k)$ represent the individual colour of element a_i,in which , f_k represents the colour of the element $k.F(A)=(F_1, ..., F_i, ..., F_k)$ represents the unified colours of polychromatic sets, in whichF_k represents the kth unified colour of polychromatic sets.

In polychromatic sets, the author uses the bool matrix$A \times F(a)$ to describe the relationship among all elements of sets and their property. That is

$$\|c_{ij}\|_{A,F(a)} = [A \times F(a)] = \begin{bmatrix} f_1 & \cdots & f_j & \cdots & f_q \\ c_{11} & \cdots & c_{1j} & \cdots & c_{1q} \\ \vdots & & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{iq} \\ \vdots & & \vdots & & \vdots \\ c_{n1} & \cdots & c_{nj} & \cdots & c_{nq} \end{bmatrix} \begin{matrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{matrix} \tag{1}$$

In which, if $f_j \in F(a_i)$, $c_{ij}=1$, or $c_{ij}=0$.The ith row of bool matrix represents

the individual colour of element a_i. The relationship between an element and a unified colour can be described by the bool matrix $A \times F(A)$. That is $\|c_{ij}\|_{A,F(A)} = [A \times F(A)]$, ,if $F_j \in F(a_i)$, $c_{ij}=1$, or $c_{ij}=0$.

The relationship between the individual colour and unified colour can be described by the bool matrix $\|c_{ij}\|_{F(a),F(A)} = [f(a) \times F(A)]$, if the existence of unified colour F_j is effected by the individual colour f_i, $c_{ij}=1$, or $c_{ij}=0$.

The polychromatic sets hierarchical structure is one particular style of polychromatic sets. It does not only combine the top-down decomposition of the traditional hierarchical structure (which can show the direct decomposition relationship among neighbouring nodes) but also describes the indirect relationship of nodes with the edge between the level and his adjacent level [12,13] .The author describe this hierarchical structure as $G^* = (A^*, C^*)$, in which, the node sets A^* represent the objects and their properties, and they can decompose until the project can be realized with project units, and the edge sets C^* represent the different constraint relationship of nodes. In fact, the sets of edges can make up the autocorrelation matrix C^* of node sets, that is $C^* = [C_{ij}] A^*$, $A^* = [A^* * A^*]$, $i > j$. In practice, the author can build the constraint matrix to describe the restriction relationship between the levels and nodes. In the model of polychromatic sets hierarchical structure, the author merged the function and method into one node in the same level, if the method can realize this function in one level ,and describe them as the order sets $< F(A(k,i_k,j_{k-1})), A(k,i_k,j_{k-1}) >$, in which, $A(k,i_k,j_{k-1})$ represents the i_kth node in the level$k$; its father node is the node of thej_{k-1}th in level $k-1$,and the colour of node $A(k,i_k,j_{k-1})$ represent its the parameter and property , which is described as $F(A(k,i_k,j_{k-1}))$. The hierarchical structure is shown as Figure 1.
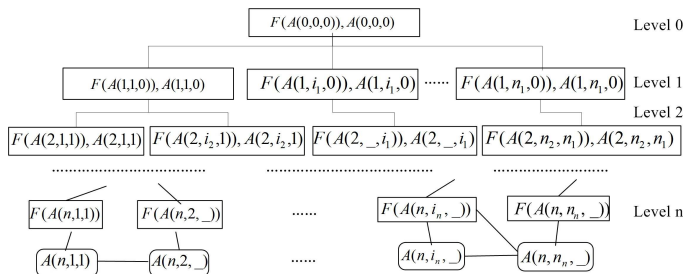

Fig. 1. The order Polychromatic sets in hierarchical structure

The recurrence formula of design object can be expressed as follows, according to the Figure 1.that is

$$< F(A(0,0,0)), A(0,0,0) >= \bigcup_{i_1=1}^{n_1} < F(A(1,i_1,0)), A(i,i_1,0) > \forall k \in [1,n], \quad (2)$$

$$< F(A(k,i_k,j_{k-1})), A(k,i_k,j_{k-1}) >= \bigcup_{i_{k+1}=m_0(k)+1}^{m_n(k)} < F(A(k+1,i_{k+1},i_k)), \quad (3)$$

$$n_k = \sum_{i_{k-1}=1}^{n_{k-1}} n(k, i_{k-1})n_0 = 1 \qquad (4)$$

$$m_0(k) = \sum_{i=1}^{i_k-1} n(k+1, i) \qquad (5)$$

$$m_n(k) = m_0(k) + n(k+1, i_k) \qquad (6)$$

In the above formula, $A(0,0,0)$is the root node; $F(A(0,0,0))$is its colour. $<$ $F(A(0,0,0)), A(0,0,0) >$ is their order sets. The $n(k+1, i_k)$ is the number of child nodes of the level k and node i_k . The $n_k$is the number of all the nodes of the level$k$.

3. Building the model of product family based on polychromatic sets hierarchical structure

3.1. The theory of mass customization model based on the polychromatic sets

The purpose of mass customization has received a quick response and met the customers' personnel needs, and improved the flexibility and efficiency of production. Normally, the mass customization includes three factors: the production efficiency must be up to the standard of mass production; the design and production process must be flexible, and in other words, the design and production process should be based on the modular and standardization; the target of production must meet customers' personal requirements. In the circumstances of mass customization, the objects of the product design are aimed at the product family; it has changed from a single-oriented product to series-oriented one. The model of product family is the key technology of mass customization. So the model of product family is a complex macrocosm, and it consists of the function model and the parts of product. So this paper uses the hierarchical structure to describe the product family, which can be shown as Figure 2.

According to the theory of polychromatic sets hierarchical structure, the basic elements of product—parts are regarded as the elements of polychromatic sets A, named a_i,and $F(A(0,0,0))$ is regarded as the unified colour of Polychromatic sets, named the product or product system;$F(A(k, i_k, j_{k-1}))$ is regarded as the individual colour, namely the function parts of product family. The property of elements, which are in the neighbour level, namely $F(A(k, i_k, j_{k-1}))$and$F(A(k+1, i_{k+1}, j_k))$, is described and computed by bool matrix$\|c_{ij}\|_{F(a), F(A)}$=$[F(a) \times F(A)]$, which describes the relationship between the individual colour sets and the unified colour sets. While $A(k, i_k, j_{k-1})$ A, the relationship between$F(A(k, i_k, j_{k-1}))$and$A(k, i_k, j_{k-1})$, which describes the property of elements and itself separately, can be described and calculated by matrix$[A \times F(A)]$.$F(A(k, i_k, j_{k-1}))$and $F(A(k, p_k, q_{k-1}))$, namely the property of different elements in the same level ,their relationship can be described and calculated by autocorrelation matrix $[F(A) \times F(A)]$.$A(k, i_k, j_{k-1})$ and

$A(k, p_k, q_{k-1})$,namely the project unit in same level , and their relationship can be described and calculated by the autocorrelation matrix $[F(a) \times F(a)]$.

Meanwhile, the different constraint relationship among the nodes of hierarchical structure can make up the autocorrelation matrix of node sets, C*=$[C_{ij}]$ A^*,A*= [A*×A*], $i > j$.In practice, the author can build the constraint matrix in order to restrict the different levels and nodes, according to their constraint relationship.
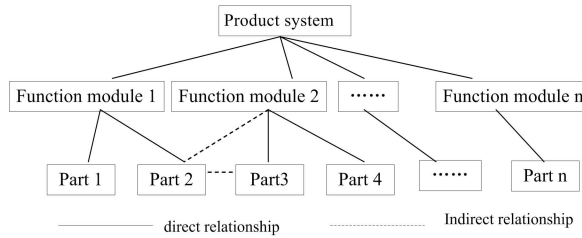

Fig. 2. The graph of product family hierarchical structure

3.2. The rebuilt method of mass customization

The aim of mass customization is to configure different function modules rapidly with customers' different requirements. According to the above theory, the step of modelling is as follows:

Step1 builds the hierarchical structure. Build the relationship among customers' requirements, product modules and parts. The structure of hierarchical is shown as Figure 2

Step 2 builds the reasoning matrix and constraint matrix. Build the customer requirement matrix, module property matrix and parts matrix separately according to the Figure 2, and analyze the potential constraint relationship between the customers' requirements and the parts, and build the constraint matrix.

Step 3 searches the projects. We can get the parts sets by calculation, according to customers' requirement. The flow chart is shown as Figure 3.

4. Application verification

The identification system is widely applied. Its module of function and parts are easy to be customized. And the author cites the example of the identification system; we explain the steps and methods of the applied polychromatic sets hierarchical structure.

The identification system can be divided into three function modules: identifying module, data storage module and data processing module. Among the modules, the identifying module has four mode, such as the fingerprint identification, iris recognition and identity identification, face recognition; the data storage mode include the SD card, WIFI, RAM, wired network; The data processing mode include IC and CPU. Shown as the Figure 4, it describes the process of decomposition from customer's requirements to the property of product module to parts, and uses the
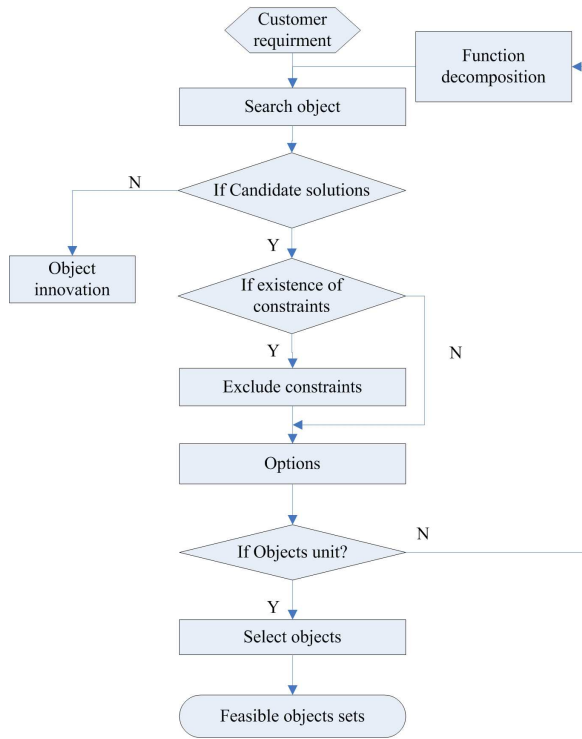
Fig. 3. The flow chart of rebuilding for Mass customization

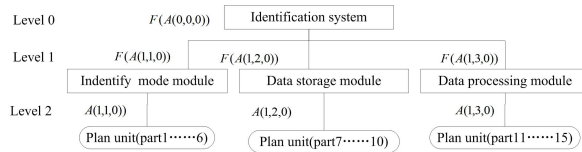edge to explain the constraint relationship of nodes.

Fig. 4. Formal structure model of Identification system

In level 0, the customer's requirements can be described as $F(A(0,0,0)) = F_1^0 F_2^0 F_3^0 F_4^0 F_5^0 F_6^0 F_7^0 F_8^0$. In the level 1, the property of function module of product family can be described as $F(A(1,1,0)) = F_1^1 F_2^1 F_3^1 F_4^1$, $F(A(1,2,0)) = F_5^1 F_6^1 F_7^1 F_8^1$, and $F(A(1,3,0)) = F_9^1 F_{10}^1$; in level 2, the parts of product family can be described as $A(1,1,0) = a_1 a_2 a_3 a_4 a_5 a_6, A(1,2,0) = a_7 a_8 a_9 a_{10}$ and $A(1,3,0) = a_{11} a_{12} a_{13} a_{14} a_{15}$. The meaning of the function unit in each level is shown as Table 1.

Table 1. The meaning of function unit in each level

Function unit	Meaning	Function unit		Meaning
F_1^0	Response time 15fps	Identify mode module	F_1^1	Fingerprint identification,
F_2^0	Response time 15fps		F_2^1	Iris recognition
F_3^0	Data storage 1000 persons		F_3^1	Face recognition;
F_4^0	Data storage 1000 persons		F_4^1	Identity identification
F_5^0	Identify distance 10"	Data storage module	F_5^1	SD card
F_6^0	Data storage 10"		F_6^1	Wired network
F_7^0	outdoors		F_7^1	WIFI
F_8^0	office		F_8^1	RAM
		Data processing module	F_9^1	CPU
			F_{10}^1	IC

Reasoning matrix: it describes the process of decomposition from customers' requirements (level zero) to the function module of product family (level 1), and the author can build reasoning matrix with $[F(a) \times F(A)]$ or $[A \times F(A)]$, i.e.

$$T_1 = [F(A(1,1,0)) \times < F(A(0,0,0)), A(0,0,0) >]$$

$$T_2 = [F(A(1,2,0)) \times < F(A(0,0,0)), A(0,0,0) >]$$

$$T_3 = [F(A(1,3,0)) \times < F(A(0,0,0)), A(0,0,0) >]$$

$$T_4 = [A(1,1,0) \times < F(A(1,1,0)) >]$$

$$T_5 = [A(1,2,0) \times < F(A(1,2,0)) >]$$

$$T_6 = [A(1,3,0) \times < F(A(1,3,0)) >]$$

Constraint matrix: There is a constraint relationship between the project unit $A(1,2,0)$ and project unit $A(1,3,0)$, so the author can build the constraint matrix according to $[F(a) \times F(a)]$, i.e. $S_1 = [A(1,2,0) \times A(1,3,0)]$.

In order to describe it conveniently, the seven matrixes above mentioned can be drawn in one map, shown as Figure 5.

If the customer's requirements for a project include the following factors(the response time is less than 15fps, the data storage is more the 1000 persons, and the distance of getting data is more 10"), the customer's requirements can be described by contour.
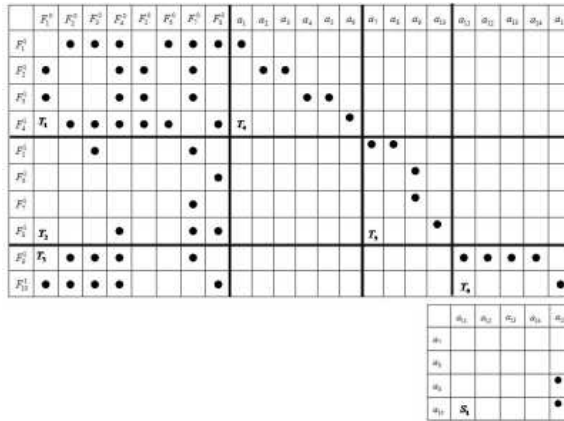
Fig. 5. Reasoning matrix and constraint matrix

F_1^0	F_2^0	F_3^0	F_4^0	F_5^0	F_6^0	F_7^0	F_8^0
	.	.		.			.

According to the reasoning matrix T_1, T_2 and T_3. We can get 4 plans; it can be described as follows:

	F_1^1	F_2^1	F_3^1	F_4^1	F_5^1	F_6^1	F_7^1	F_8^1	F_9^1	F_{10}^1
Plan1				.	.	.			.	
Plan2				.	.	.				.
Plan3				.	.			.	.	
Plan4				.	.			.		.

5. Conclusion

The paper promotes a formal method of the mass customization based on the polychromatic sets hierarchical structure. The method describes the mapping modules among the customers' requirements, the product family modules and parts, and also illustrates the constraint relationship directly and indirectly. By searching the reasoning matrix, the author of the paper can get feasible resolutions for the mass customization, and in the circumstances of constraint matrix, the most reasonable plan can be obtained from the resolutions mentioned above. Meanwhile, the bool matrix built based on this system method is simpler and easier to achieve the formal representation and programming.

References

[1] Q. Lei, X. W. Li, W. F. Guo: *Process sequence planning modeling based on polychromatic sets.* International journal of grid and distributed computing *9* (2016), No. 9, 155–172.

[2] G. Yang, K. W. Chen: *A scheme design and configuration model of mechanical equipment processing technology based on polychromatic sets theory.* International journal of control and automation *7* (2014), No. 11, 137–144.

[3] Z. J. Xu, Y. Li. J. Zhang: *A dynamic assembly model for assembly sequence planning of complex product based on polychromatic sets theory.* Assembly automation *32* (2012), No. 2, 152–162.

[4] X. Q. Gao, L. D. Xu, X. P. Wang: *Workflow process modelling and resource allocation based on polychromatic sets theory.* Enterprise information systems *7* (2013), No. 2, 198–226.

[5] X. H. Wang, S. C. Li: *Scalable routing modelling for wireless ad hoc networks by using polychromatic sets.* IEEE Systems Journal *7* (2013), No. 1, 50–58.

[6] X. Q. Gao, X. P. Wang, Y. Li: *Workflow dynamic change and instance migration approach based on polychromatic sets theory.* International journal of computer integrated manufacturing *29* (2016), No. 4, 386–405.

[7] S. Timothy, B. Aaron, S. Laur: *From user requirements to commonality specifications: An integrated approach to product family design.* Research in engineering design *23* (2012), No. 2, 141–153.

[8] X. G. Luo, W. Li, C. K. Kwong: *Optimisation of product family design with consideration of supply risk and discount.* Research in engineering design *27* (2016), No. 1, 37–54.

[9] C. M. Wang, F. S. Huang: *A method of coordinate measuring path planning based on polychromatic sets.* Applied mechanics and materials (2016) 599–601.

[10] J. X. Zhang, Y. J. Zhou, Q. Zhang: *Product conceptual design based on polychromatic sets theory and axiomatic design.* Lecture notes in electrical engineering *367*, (2016) 1061-1069.

[11] Y. X. Wang, J. Wei, C. C. Gao: *Customization design of cloud manufacturing resources based on polychromatic sets theory.* Proceedings - 2015 7th international conference on intelligent human-machine systems and cybernetics *1* (2015) 518–521.